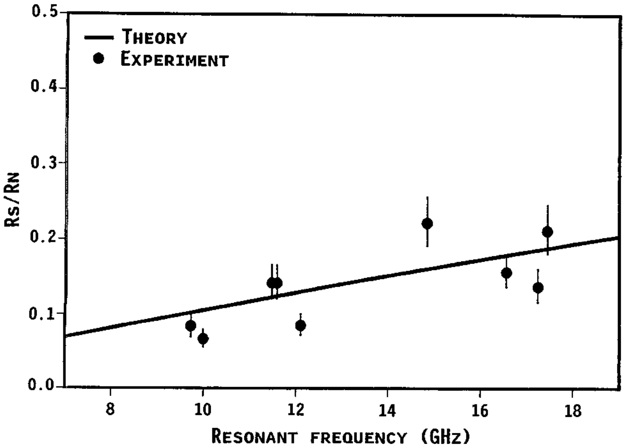

Fig. 4. Comparison of normalized surface resistance data with theory.

the data points were obtained by assuming an error of 1.7% in the S_{21} measurements. This error is due to the return loss of the coaxial to waveguide transitions, which is neglected in the impedance extraction method. A $VSWR$ of 1.3 in the transitions will cause an error of 1.7% in S_{21} measurements.

IV. Conclusions

This paper reports a novel technique for measuring the resistive properties of high-temperature superconductors. It utilizes an analysis developed by Eisenhart for a two-gap, electrically isolated resonant strip in waveguide. Results of normalized surface resistance measurements show good agreement with a modified form of the Mattis–Bardeen extension of the Bardeen, Cooper, Schrieffer theory of superconductivity.

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Modal Analysis of Open Groove Waveguide

SAMIR F. MAHMOUD, SENIOR MEMBER, IEEE

Abstract —**A self-contained analysis for lower order modes in the open groove waveguide is presented. By adopting an approximate form for the fields in the outer region of the guide and then performing the remainder of the analysis rigorously, closed-form results for the fields and modal equation are obtained. The analysis allows a simple transverse network representation which can be compared with that obtained by Oliner and Lampariello. Universal dispersion curves, obtained numerically for the dominant mode, are presented.**

I. Introduction

The open groove guide has long been recognized as a low-loss waveguide which is most suitable for millimeter-wave circuits [1]–[5]. The low loss property is attributed to the absence of any dielectric material and to the low conductor losses, a condition which stems from the electric field lines of the dominant mode being parallel to the guide walls. Referring to Fig. 1(a), the electric field of the dominant mode is mainly parallel to the sidewalls. It is oscillatory in the inner region, $|y| < b/2$, and decaying in the outer regions, $|y| > b/2$. Because of the junction discontinuity at $y = \pm b/2$, a rigorous analysis of modes requires an expansion of the fields in terms of transverse modes in both the inner and outer regions. The process is lengthy [4] and the mathematical complexity will obscure any physical insight into the mode behavior. On the other hand, simple approximate solutions based on a single transverse mode field representation in each region (e.g. [1], [3]) will not have sufficient accuracy. More recently Oliner and Lampariello [6] have presented a simple and yet accurate solution based on an equivalent transverse network, taking into account the effect of higher order modes generated at the junction planes $y = \pm b/2$. Thus, the inner region is represented by a transmission line (Fig. 1(b)) connected through a step transformer to another line which represents the outer region. The effect of the higher order transverse modes is lumped into a susceptance B. By adopting results from [7], Oliner *et al.* [6] have deduced a useful formula for the susceptance B. They verify their results by comparing them with previous experimental data obtained by Nakahara and Kurauchi [1] and report favorable comparisons.

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The author is with the Electrical and Computer Engineering Department, Kuwait University, P.O. Box 5969, Safat 13060, Kuwait

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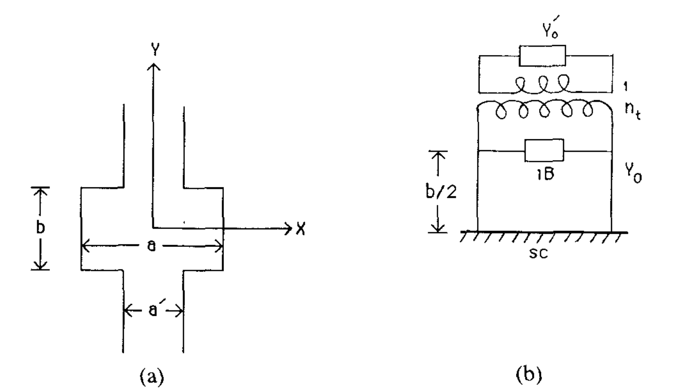

Fig. 1 The open groove guide (a) Geometry. (b) Transverse equivalent network for even modes.

In the present paper, we try to give a rather self-contained formulation to the lowest order modes of the groove guide. A simplifying assumption is adopted at the outset of the analysis whereby a simple field distribution is assumed in the outer regions. Apart from this assumption, the rest of the analysis is quite rigorous and leads to a simple modal equation. The latter can be interpreted in terms of a transverse network representation whose parameters are expressed in closed form.

II. Analysis

Since the groove guide is homogeneously filled, the basic modes are either TE or TM to z (Fig. 1(a)). The dominant mode is the TE_{10} mode, resembling the same mode in a closed rectangular waveguide. So, in the following we consider TE_{1n} modes. The region $|y| > b/2$ is assumed to be sufficiently long, so that all fields decay to negligible values before reaching the open end. For TE_{1n} modes all fields are expressible in terms of the h_z field component while $e_z = 0$.

Apart from a common field dependence $\exp(i\omega t - i\beta z)$ we start by approximating the field in the grooved region $|y| \geqslant b/2$ by

$$h_z \simeq \sin(\pi x/a')\exp(-\alpha(|y| - b/2)) \quad (1)$$

while in the central region, $|y| \leqslant b/2$, a general field expression is used:

$$h_z = \sum_{m=0}^{\infty} A_m \sin[(2m+1)\pi x/a]\cos k_{ym} y \quad (2)$$

where only even modes about $y = 0$ are considered and

$$k_{ym}^2 + [(2m+1)\pi/a]^2 = (\pi/a')^2 - \alpha^2 = k_0^2 - \beta^2. \quad (3)$$

Both (1) and (2) satisfy the wave equation and the boundary conditions at the sidewalls. Thus it remains to satisfy the continuity of h_z, e_x, and h_x at the plane $y = b/2$. These three boundary conditions reduce to two since the continuity of h_z at all values of x implies the continuity of h_x, which is proportional to $\partial h_z/\partial x$. Therefore, it is sufficient to apply the boundary conditions to h_z and e_x only.

Continuity of h_z reads

$$\sum_{m=0}^{\infty} A_m \sin[(2m+1)\pi x/a]\cos(k_{ym}b/2) = \sin(\pi x/a') \qquad \text{for } |x| \leqslant a'/2 \quad (4)$$

and continuity of e_x reads

$$\sum_{m=0}^{\infty} k_{ym} A_m \sin[(2m+1)\pi x/a]\sin(k_{ym}b/2) = \begin{cases} \alpha\sin(\pi x/a') \cdots & \text{for } |x| < a'/2 \\ 0 & \text{for } a/2 \geqslant |x| \geqslant a'/2. \end{cases} \quad (5)$$

Thus while the electric field e_x is defined over the entire interval $(-a/2, a/2)$, the magnetic field h_z is defined only in the range $(-a'/2, a'/2)$ and is unknown outside. It is then appropriate to try to expand h_z in terms of a complete set of functions inside the region $(-a'/2, a'/2)$. We choose the set of functions $\sin[(2n+1)\pi x/a']$, $n = 0, 1, \cdots$ for this purpose. On the other hand e_x is expanded in terms of the set of functions $\sin[(2n+1)\pi x/a]$ with range $(-a/2, a/2)$. To implement these ideas, multiply (4) by $\sin[(2n+1)\pi x/a']$ and integrate between the limits $-a'/2$ and $a'/2$:

$$\sum_{m=0}^{\infty} A_m c_{mn} \cos(k_{ym}b/2) = (a'/2)\delta_{n0} \quad (6)$$

where $\delta_{n0} = 0$ for $n \neq 0$ and $= 1$ for $n = 0$, and

$$c_{mn} = \int_{-a'/2}^{a'/2} \sin[(2m+1)\pi x/a]\sin[(2n+1)\pi x/a']\,dx. \quad (7)$$

Similarly, multiply (5) by $\sin[(2n+1)\pi x/a]$ and integrate over the range $(-a/2, a/2)$:

$$A_n k_{yn}(a/2)\sin(k_{yn}b/2) = c_{n0}\alpha. \quad (8)$$

Substituting for A_m in (6) from (8) and putting $n = 0$ in the former equation, it becomes

$$\sum_{m=0}^{\infty} (c_{m0})^2 \cot(k_{ym}b/2)/k_{ym} = (a'a/4\alpha). \quad (9)$$

This is the modal equation for the TE_{1n} modes, since the only unknown in the equation is β. This can be written in a form that allows a transverse circuit representation as follows:

$$-iY_0\cot(k_{y0}b/2) - iY_0 \sum_{m=1}^{\infty} (c_{m0}/c_{00})^2 \cot(k_{ym}b/2)k_{y0}/k_{ym} + Y_0'/n_t^2 = 0. \quad (10)$$

The first term in this equation is the input admittance of a short-circuited transmission line of length $b/2$ and characteristic admittance $Y_0 = \text{constant } K/k_{y0}$, representing the central region. The third term, representing the grooved region, is the reflected admittance of an infinite line of characteristic admittance $Y_0' = K/(-i\alpha) = iK/\alpha$ through a step-up transformer of turns ratio n_t (see Fig. 1(b)). Finally the second term is a shunt admittance $= iB$ that accounts for the higher order modes generated at the junction $y = b/2$. By comparing (9) with (10), we get

$$n_t = 2c_{00}/(aa')^{1/2} = (4/\pi)(a'a)^{3/2}\frac{\cos(\pi a'/2a)}{1-(a'/a)^2} \quad (11)$$

where (7) has been utilized to get c_{00}. Finally, the summation term in (9) accounts for higher order modes and is equal to $-B/Y_0$, which is obviously dependent on the modal phase constant β through the factor k_{ym}. However in a practical case and for low-order modes, it is appropriate to approximate k_{ym}, $m \geqslant 1$, by (see (3))

$$k_{ym} = \left[k_0^2 - \beta^2 - (2m+1)^2\pi^2/a^2\right]^{1/2} \simeq i(2m+1)\pi/a.$$

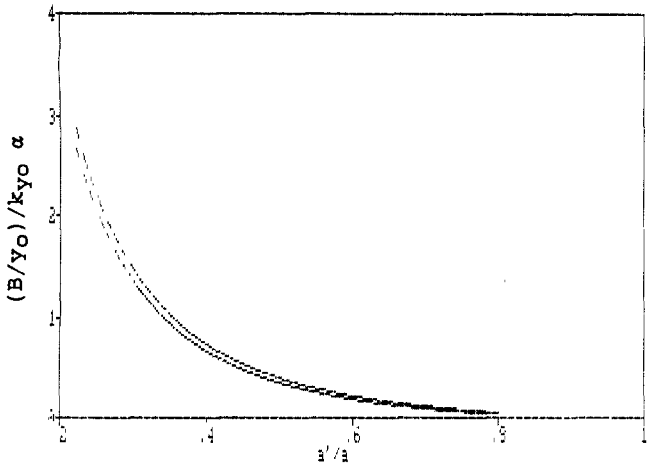

Fig 2. Plots of $(B/y_0)/k_{y0}\alpha$ Curve *1*: eq. (13). Curve *2*: eq. (14) (from [6]).

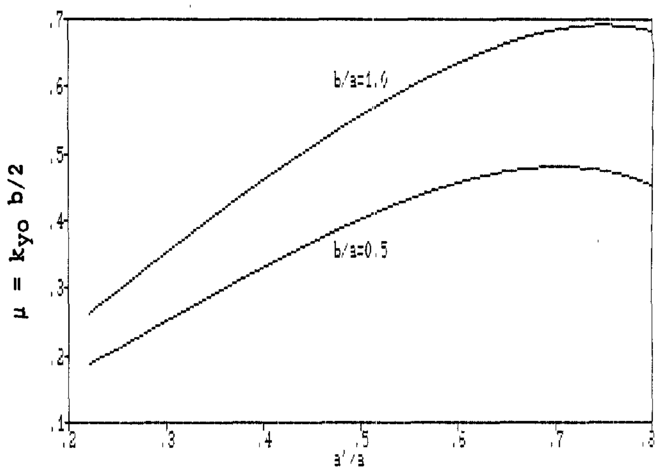

Fig. 3. Eigenvalue $u = k_{y0}b/2$ for the dominant mode versus (a'/a) for $b/a = 1/2$ and 1.

This renders B/Y_0 independent of β. Therefore

$$B/Y_0 \simeq (k_{y0}a) \sum_{m=1}^{\infty} (c_{m0}/c_{00})^2 \cdot \coth[(2m+1)\pi b/2a]/(2m+1)\pi \quad (12)$$

and is dependent only on the geometry.

The $\coth[\cdot]$ term in this expression accounts for the coupling between evanescent fields at the two junctions $y = \pm b/2$. Whenever b is sufficiently large, (12) is further simplified by setting the $\coth[\cdot]$ term equal to unity. This requires that $3\pi b/a > 4$, say, or $b/a > 0.42$. Under this condition,

$$(B/Y_0) \simeq (k_{y0}a) \sum_{m=1}^{\infty} (c_{m0}/c_{00})^2/(2m+1)\pi$$

$$= (k_{y0}a) \sum_{m=1}^{\infty} \frac{(1-a'^2/a^2)^2 \cos^2[(2m+1)\pi a'/2a](2m+1)/\pi}{[1-(2m+1)^2 a'^2/a^2]^2 \cos^2(\pi a'/2a)}. \quad (13)$$

Now, this can be compared with the expression obtained by Oliner *et al.* [6] for the same quantity; namely B/Y_0 in [6] is expressed by

$$B/Y_0 = (k_{y0}a) \cdot 0.55(2/\pi) \cot^2(\pi a'/2a). \quad (14)$$

Finally, the modal equation (10) can be recast in the simpler form

$$\cot(k_{y0}b/2) - B/Y_0 - (k_{y0}/\alpha)/n_t^2 = 0 \quad (15)$$

where B is given by (13), n_t by (11), and α is related to k_{y0} by (3).

III. Numerical Results

First we compare expressions (13) and (14) for B/Y_0. To do so, we compute the quantity $(B/Y_0)/(k_{y0}a)$, which is a function of (a'/a) only, using both equations. This is plotted in Fig. 2 and it is clear that the two expressions are very close to each other, although each is obtained by different means. Next, we solve the modal equation (15) in terms of $u = k_{y0}b/2$ and plot this for the dominant mode (lowest u) versus (a'/a) for $b/a = 1/2$ and 1 in Fig. 3. The cutoff wavenumber k_c is obtained from (3) by setting $\beta = 0$; namely $k_c^2 = (2u/b)^2 + (\pi/a)^2$.

IV. Conclusions

We have tried to present a self-contained analysis for the lower order modes in the open groove waveguide based on a simplifying assumption used right at the outset. Thus, the modal equation (10) has been derived for the TE_{1n} modes by adopting a simple field variation in the groove region. The modal equation (10) is interpreted in terms of the transverse network suggested by Oliner *et al.* [6], and shown in Fig. 1(b), where the junction susceptibility B is generally dependent on the longitudinal phase constant β and the geometrical factors a, (a'/a), and b/a. However, for the lower order modes, B can be considered independent of β, and is then given by (12). The dependence of B on the ratio (b/a) reflects the coupling between the evanescent transverse modes at the two junctions $y = \pm b/2$. It has been shown, however, that if $b/a \geqslant 0.42$ the dependence on this ratio becomes very weak. The reduced expression (13) for B is then shown to agree numerically with that derived by Oliner *et al.* [6]. The modal equation (10) or (15) is believed to provide a simple and accurate means of characterizing the dominant mode, and perhaps the next few low order modes of the TE_{1n} type in the open groove guide. A bonus of the present analysis is the provision of a simple expression for the modal fields, which are given by (1) and (2), with A_m given explicitly by (8). Thus it is a simple exercise to compute the power flow and the conductor losses for the modes of interest.

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